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EXPERIMENTAL AND COMPUTATIONAL INVESTIGATION OF HEAT FLUXES IN A SUPERSONIC DIFFUSER

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The internal structure of a turbulent flow in a boundary layer brings about an expressed two-layer character of flow with respect to the characteristic relaxation time of perturbations. The different relaxation times of perturbations in the internal and external parts of a boundary layer are not taken into account by the Boussinesq hypothesis, in which turbulence is determined by the usual law for a viscous liquid of dependence of stresses on the strain rate, which differs only in the renormalized viscosity coefficient [1].

It was proposed to take account of the specific influence of the internal flow structure on the relaxation of perturbations using the Clauser parameter $\beta = (\delta^* dp/dx)/\tau_w$ (dp/dx is the longitudinal pressure gradient, and δ^* and τ_w are the displacement thickness and friction near the solid surface). The boundary layers in which $\beta =$ const are called equilibrium boundary layers, according to Clauser, and those in which $\beta \neq$ const, nonequilibrium. Various attempts have been made to overcome the restrictions of the Boussinesq formula for calculating nonequilibrium boundary layers [1], in particular, by modifying the coefficient of turbulent viscosity ν_t [2, 3]. In calculations of heat exchange in supersonic flows with separation zones the relations accounting for the elevated level of turbulence in nonequilibrium layers were used [4, 5]. However, all these approaches, which consider the extrinsic aspect of the above phenomenon, have a number of limitations in applications, especially for heat exchange predictions.

Models that allow for the orientational interaction of the vectors of the internal angular momentum and external force fields [6] make it possible to understand some reasons for the uniqueness of the vortex structure of turbulence [7] and simulate its influence in heat exchange calculations.

A supersonic diffuser is one of the most thermally stressed elements of the various devices of aviation and rocket equipment. Owing to its small relative length ($a/D_i \leqslant 10$, a and D_i are the length and diameter of the diffuser), the presence of shock waves, and the formation of local separation zones, the turbulent boundary layer at the diffuser internal surfaces is essentially nonequilibrium.

In the present work we present the results of an experimental study of a flow and heat exchange inside a subsonic diffuser, as well as a calculation of heat exchange coefficients along its generatrix on the internal surface. In contrast to [4, 5], the heat exchange in a nonequilibrium turbulent boundary layer is simulated on the basis of the considerations of nonequilibrium turbulence [7] using a specially constructed function that allows for the orientational effect of the main flow on large-scale turbulence. This function does not contain additional empirical constants and, therefore, agrees with the main idea of the asymptotic theory [8] of the relative law of heat exchange.

1. Experimental Investigation. Experiments were carried out on a large-scale gasdynamic ramjet device designed for modeling internal processes in rocket engines that realizes large Reynolds numbers at air flow rates of up to 10 kg/sec.

The model presented in Fig. 1 is that of a nozzle–diffuser system. A supersonic conical Laval nozzle 1 with a nozzle throat area of $11.58 \cdot 10^{-4}$ m^2 (d_+ is the diameter of the nozzle throat in the section A–A) and outer diameter $d_- = 0.104$ m is joined to a cylindrical tube 2 with diameter $D_2 = 0.14$ m (or $D_1 = 0.125$ m, which is shown by dashed contour 3 on the inner surface) and length $a = 0.75$ m.

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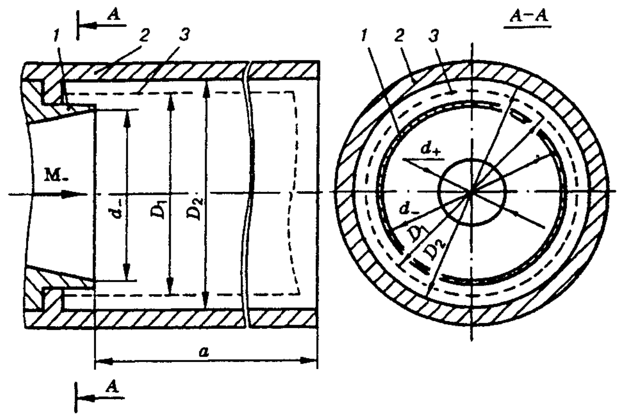

Fig. 1

A diffuser diameter D_i greater than the nozzle outlet diameter $D_i = (1.05\text{–}1.5)\, d_-$ was used for nozzle operation as an ejector. The angular deviation of the nozzle axis from the diffuser axis was excluded. The Mach number of the gas issuing from the nozzle was $\mathrm{M}_- = 3.6$; total pressure at the nozzle inlet section was $p_+^* = (31\text{–}41) \cdot 10^5$ Pa.

The pressure fields across the model channel were measured by a rake of 17 total-pressure probes. The diameter of the Pitot hole was $1 \cdot 10^{-2}$ m. The distance between the axes of the holes varied from $5 \cdot 10^{-3}$ m at the center of the rake to $1 \cdot 10^{-2}$ m at its edges. To measure pressure at the inner channel wall along the generatrix of the surface we used a diffuser model with 79 pressure-tap holes $0.5 \cdot 10^{-3}$ m in diameter placed at a distance of $(3\text{–}10) \cdot 10^{-3}$ m from each other. To determine the pressure fields and distributions at the channel surface we used integral silicon pressure gauges providing accuracy of up to 0.5%.

The technique of measuring local coefficients of heat exchange intensity is described in detail in [9]. To measure the temperature along the generatrix (inside a cylindrical diffuser) 141 Chromel–Copel thermocouples $(0.1 \cdot 10^{-3}$ m) were embedded into a textolite surface at a distance of $(2\text{–}5) \cdot 10^{-3}$ m from each other. The random error in measuring the heat exchange coefficients did not exceed 5%; the total error could reach 15% in the sections with considerable thermal nonuniformity due to heat losses into the wall.

An instrumental complex consisting of a MERA60-45K microcomputer, an "Analog-1" automatized multi-channel measuring system, a Shch1516 digital dc voltmeter, and F799/1 switches enabled us to perform measurements in the necessary real time, to register the e.m.f. of the thermocouples, and to process the signals from the pressure gauges.

The character of change of the profiles of the Mach number M normalized with respect to the Mach number M_- when the nozzle is connected to a diffuser of diameter D_2 is presented in Fig. 2. The M/M_- profiles (points 1–5) correspond to the cross-sections of the inner part of the diffuser with coordinates for $x = 0.05$, 0.15, 0.4, 0.65, and 0.7 m. Here x is reckoned from the nozzle exit.

The velocity fields inside the diffuser were calculated using the Rayleigh formula [10]

$$\frac{p_0'}{p_0} = \frac{\dfrac{2æ}{æ+1}\mathrm{M}^2 - \dfrac{æ-1}{æ+1}}{\left(\dfrac{4æ}{(æ+1)^2} - \dfrac{2(æ-1)}{(æ+1)^2}\dfrac{1}{\mathrm{M}^2}\right)^{æ/(æ-1)}\left(1 + \dfrac{æ-1}{2}\mathrm{M}^2\right)^{æ/(æ-1)}},$$

which connects the total pressure p_0' measured by the probe behind the shock and the total pressure p_0 ahead of the shock wave. Here $æ = 1.4$ is the adiabatic exponent for air.

In this diffuser type, a flow with reflected oblique shock waves is realized without a subsequent normal shock wave due to the low efficiency of the shortened diffuser, where $a/D_2 = 5$. It is well known that the process of flow development in a nozzle–diffuser system beginning from the nozzle throat to the exit section

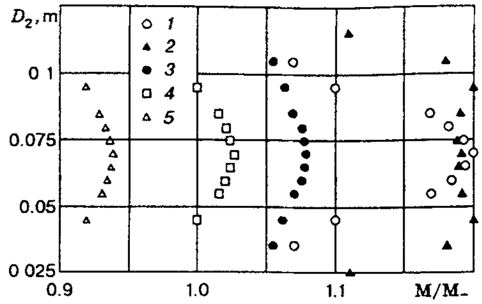

Fig. 2

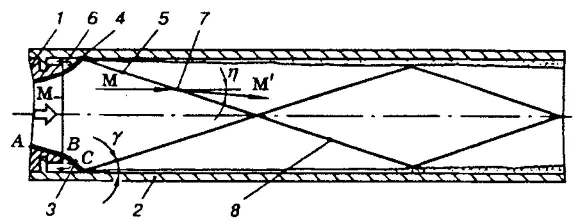

Fig. 3

of the diffuser proceeds in several stages successively. The stages involve: triggering of the nozzle, which is characterized by a nonstalling flow in the nozzle; attachment of a jet boundary layer to the diffuser walls (triggering of the diffuser); rearrangement of the system of shocks in the flow core and attachment of the shocks in the attachment regions; and stabilization of the Mach number fields along the entire route (Fig. 2). Figure 3 shows the part of the nozzle exit 1 and starting section of the diffuser 2 with D_2, inside which the gas flow under study is presented schematically. A jet shear layer 3 flows on the diffuser walls in the attachment region 4 during the efflux from the supersonic nozzle. In the vicinity of region 4, a shock 5 is formed with slope angle γ, and a reverse flow is formed in region 6. After the interaction between the jet with velocity corresponding to the Mach number M and the oblique shock front (point 7) the jet is deflected by angle η, and the Mach number decreases to the value M′. Then repeated interactions between the shocks and the turbulent boundary layer occur due to their reflection from the diffuser walls.

Smoothing of the fields of the Mach numbers (see Fig. 2), as the gas moves from the nozzle exit to the exit section of the diffuser, is characterized by a decrease in the spread of the Mach-number ratios M/M_- in the flow core during transition from the initial section (points 1) to the final section under study (points 5) by more than a factor of 6.

The character of the pressure distribution for a diffuser with cross-section diameter D_1 is presented in Fig. 4a. One can see the features of separation of the approaching boundary-layer jet in the vicinity of the divergence angle ABC (see Fig. 3) and attachment at the point $z = (x - l)/d_-$ (x is the length from the nozzle throat to the current section of the diffuser, l is the length of the section from the nozzle throat to the nozzle exit, and d_- is the diameter of the nozzle exit) with the peak value $p/p_- = 1.7$ ($z = 0.5$). At the point $z = 3.75$, $p/p_- = 1.25$, which indicates interaction between the reflected shock 8 (see Fig. 3) and the boundary layer. In the experiments a local separation zone of the order of the boundary layer thickness in size was found.

For a diffuser of diameter D_2 the character of the pressure distribution (Fig. 5a) coincides qualitatively with that presented in Fig. 4a, with differences only in the peak values p/p_- and their coordinates.

The experimental data obtained on the distribution of local coefficients of heat exchange intensity α (Figs. 4c and 5b) reflect qualitatively the main features of the flow stemming from the character of the pressure distribution. In particular, growth of the heat exchange level is observed in the attachment region of the flow up to $\alpha = 2{,}300$ W/(m$^2 \cdot$K) for a diffuser of D_1 at the point $z = 0.5$ (Fig. 4b) and up to $\alpha = 2{,}100$ W/(m$^2 \cdot$K) for a diffuser of D_2 at the point $z = 0.5$ (Fig. 5b). Behind the attachment region after the flow is turned to a horizontal direction, a sharp decrease (approximately fourfold) in the heat exchange coefficient occurs due to the transition of the flow parameters to an unperturbed state. Then, downstream, the attachment of the reflected shock 8 (see Fig. 3) and the appearance of a small separation zone influence the heat exchange coefficient, as in the case with pressure distribution.

It is evident from the research results that the most thermally stressed region is the attachment region 4 (see Fig. 3) of the jet issuing from a supersonic nozzle. There are no clearly expressed peak regions in the increase in the heat-exchange level upon interaction of the reflected shocks downstream from this section, although it is essential for an average heat-exchange level. This should be taken into account in practice when selecting an optimal heat protection for the walls of supersonic diffusers.

2. Analytical Model. The approach to the problem of calculation of heat exchange along a surface with generatrix breaks is based on the Kutateladze–Leont'ev asymptotic theory of boundary-layer turbulence [8], in which the relative variation of the integral thermal characteristics under the effect of different disturbing factors is independent of empirical constants and is not connected with any special type of semiempirical theory.

The initial system of equations for calculating the processes of heat exchange is as follows:

$$\frac{d\mathrm{Re}_T^{**}}{d\bar{x}} + \frac{\mathrm{Re}_T^{**}}{\Delta T}\frac{d(\Delta T)}{d\bar{x}} = \mathrm{Re}_L\mathrm{St}, \qquad \mathrm{St}_0 = \frac{B}{2}(\mathrm{Re}_T^{**})^{-b}\mathrm{Pr}^{-n}, \tag{2.1}$$

where $\mathrm{Re}_T^{**} = \rho_e V_e \delta_T^{**}/\mu_*$ is the Reynolds number constructed for the energy-loss thickness δ_T^{**}; $\Delta T = (T_w - T_w^*)$ is the difference between the wall and recovery temperatures; $\bar{x} = x/L$; L is the characteristic geometric size of the surface ($L = h$); $\mathrm{Re}_L = \rho_e V_e L/\mu_e$ is the Reynolds number constructed for the characteristic size of the surface L; μ_* is the characteristic dynamic viscosity at the deceleration temperature; ρ_e and V_e are the density and velocity outside the boundary layer; $\mathrm{St} = q_w/(\rho_e V_e c_p \Delta T)$ is the Stanton number; q_w is the density of the heat flux at the wall; c_p is the isobaric heat capacity; and $B = 0.0256$, $b = 0.25$, and $n = 0.75$ are the coefficients for $\mathrm{Re}_T^{**} < 10^4$.

For the heat flux density near the solid surface for an equilibrium turbulent boundary layer we write

$$q_w = -c_p\rho\,\langle v_y\Theta\rangle\, g(\Theta^2). \tag{2.2}$$

Here $g(\Theta^2)$ is the function of the effect of the density pulsations due to temperature fluctuations Θ and v_y are the velocity pulsations.

Following [7], in a nonequilibrium boundary layer the balance between the intrinsic angular momenta of large-scale turbulent vortices $\langle\mathbf{m}\rangle$ and the local observed angular momenta of the main flow $\lambda^2\nabla\times\langle\mathbf{V}\rangle$ is destroyed (λ is the characteristic scale of the vortices). As a result, additional (moment) stresses appear in such a medium, which are due to the transfer of intrinsic angular momentum. Then instead of (2.2) we have

$$q_w = -c_p\rho g\,(\Theta^2)\,(\langle v_y\Theta\rangle + f\,(\nabla\langle T\rangle, \langle\mathbf{m}\rangle, \nabla\times\langle\mathbf{V}\rangle)) = -c_p\rho g\,(\Theta^2)\langle v_y\Theta\rangle G_T,$$

where $G_T = 1 + f\,(\nabla\langle T\rangle, \langle\mathbf{m}\rangle, \nabla\times\langle\mathbf{V}\rangle)/\langle v_y\Theta\rangle$.

The function G_T can be approximately constructed, reasoning from general physical considerations of the solenoidal nature of the interaction of the vectors $\langle\mathbf{m}\rangle$, $\nabla\times\langle\mathbf{V}\rangle$, and $\nabla\langle T\rangle$.

The simplest example of similar interactions is the potential of the velocity field induced by a system of closed vortex filaments in an ideal fluid [11]:

$$\varphi = -\sum_k \frac{\Gamma_k}{4\pi}\int\limits_{\sum_k}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)d\sigma.$$

Then kinetic energy of the fluid is presented by the relation

$$\frac{1}{\rho}E = \frac{1}{2}\int\limits_V|\mathrm{grad}\,\varphi|^2 d\tau.$$

Considering that one of the main sources of the orientational effects of the main flow on large-scale turbulence in a boundary layer is the longitudinal pressure gradient, the function of the effect of the turbulent nonequilibrium state was taken as

$$G_T(x) = 1 + \left(\int\limits_0^x \frac{|(\partial p/\partial\xi)|}{(x-\xi)^{1/2}}\,d\xi\right)^2, \qquad \xi < x. \tag{2.3}$$

Here x and ξ are normalized to the characteristic size $L = h$ [$x > 1/2$, $h = (D_i - d_-)/2$]; pressure p is related to a maximum value in the calculation region. The point $x = 0$ coincides with the beginning of the dynamic

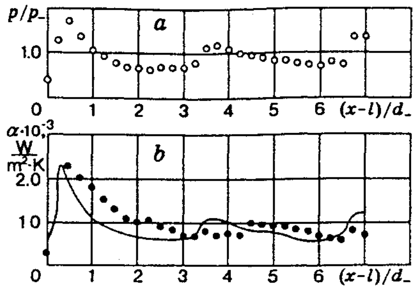

Fig. 4

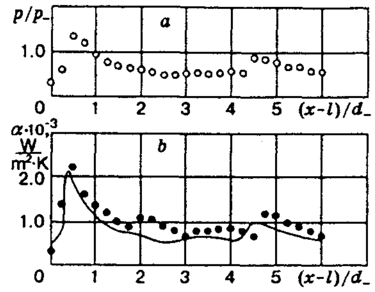

Fig. 5

boundary layer. We write the calculation algorithm of function (2.3) at any points $j = 1, 2, 3, \dots$ as

$$(G_T)_j = 1 + \Big(\sum_{i=1}^{j} \frac{|(\Delta p)_i|}{((j-i+1)\,\Delta x)^{1/2}} \Big)^2,$$

where Δx is the distance between the two closest calculation points; Δp is the variation of static pressure on a body surface section of length Δx.

The quantity St on the right-hand side of Eq. (2.1) is connected to St_0 by a limiting relation (with $\mathrm{Re}_T^{**} \to \infty$) [8], which in view of function (2.3) has the form $\mathrm{St} = \psi_T \psi_M G_T \mathrm{St}_0$. Here $\psi_T = (2/\sqrt{T_w/T_w^*} + 1)^2$ is the relative function of nonisothermicity; $\psi_M = [(\arctan M\sqrt{\chi\,(æ-1)/2})/(M\sqrt{\chi\,(æ-1)/2})]^2$ is the relative compressibility function; $\chi = \mathrm{Pr}^{1/3}$ is the recovery coefficient. The function (2.3) shows that heat exchange at each point on the surface depends on the rate of pressure change on all previous parts of the surface, beginning from the point of growth of the thermal boundary layer, while the influence of separate sections with a nonzero pressure gradient decreases with distance from them.

The determination of static pressure on the inner walls of the diffuser presents no principal difficulties. In the present work the distributions of static pressure on the diffuser walls, which are necessary for calculating the heat exchange, were taken from the obtained experimental data. A solution of Eq. (2.1) is written in the form [8]

$$\mathrm{Re}_T^{**} = \frac{1}{\Delta T} \left(\frac{B}{2} (1+b)\, \mathrm{Re}_L^* \mathrm{Pr}^{-b} \int_{\bar{x}_1}^{\bar{x}} \psi_T \psi_M G_T \left(\frac{\mu_w}{\mu_w^*}\right)^b C\,(1-C^2)^{\frac{1}{æ-1}} \Delta T^{1+b} d\bar{x} \right)^{\frac{1}{1+b}},$$

where $\mathrm{Re}_L^* = \rho^* V_{\max} h/\mu_w^*$; $c = V/V_{\max}$; $V_{\max} = \sqrt{2 c_p T^*}$; μ_w, and μ_w^* are the dynamic viscosity at the wall and recovery temperatures, respectively.

In calculating the Stanton numbers under the conditions in question we made a correction allowing for nonsimultaneous evolution of dynamic and thermal boundary layers [8]:

$$\mathrm{St} = \mathrm{St}_0 \psi_T \psi_M G_T \left(\frac{x - x_1}{x}\right)^{0.086}$$

(x_1 is the length of initial heat-insulated section).

The coefficients of heat exchange intensity α are determined from the formula [8]

$$\alpha = c_p \rho V \mathrm{St}.$$

In the calculations of heat exchange in the local separation zone, a boundary layer in the reverse flow beginning at the attachment region was considered. The reverse flow can be neglected in analyzing the pressures in the separation zones; however, it is not rewarding in a detailed consideration of the heat exchange parameters at the surface.

The algorithm which was developed and the program for calculating the heat exchange distributions are realized using an EC-1841.10 IBM-compatible personal computer. The results of calculation of the heat exchange characteristics in the context of the integral method proposed at the inner surface of a diffuser for supersonic gas flow are presented in Figs. 4b and 5b by solid curver. One can see that they are in rather good agreement with the experimental data, and the maximum value for the heat exchange level in the attachment region also correlates well with the calculated values.

Thus, the experimental studies enabled us to refine the structure of the flow inside a supersonic diffuser and determine its most thermally stressed sections. The calculated local coefficients of heat exchange intensity with allowance for the effect of the turbulent nonequilibrium state show good agreement with the experimental results.

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